# Onzièmes "Journées Besançon-Neuchâtel d'Analyse Fonctionnelle".

# Besançon, 14-16 June 2023.

# Book of abstracts

### Catalin Badea (Université de Lille)

### "How to estimate the norm of a matrix (and why it is important)"

It is quite easy to express the operator norm of a $2 \times 2$ matrix, viewed as an operator on $\mathbf{C}^2$, in terms of the entries of the matrix. In particular, the upper triangular matrix with $a$ and $b$ on the diagonal and $c$ in the upper corner has operator norm no greater than one if and only if $\max(|a|, |b|) \leq 1$ and $|c|^2 \leq (1 - |a|^2)(1 - |b|^2)$. The corresponding problems for $3 \times 3$ and higher matrices are considerably more difficult.

The aim of my talk is to discuss these problems and to present some applications. This is based on joint work with Axel Renard.

### Florent Baudier (Texas A&M University, USA)

### "Quantitative metric embeddings of trees or diamond graphs into Heisenberg groups"

In this talk I will discuss quantitative results regarding the distortion of bi-Lipschitz embeddings, and compression rate of coarse embeddings, of trees or diamond graphs into Heisenberg groups. In particular, I will focus on various point-configuration inequalities (tripod, short diagonal, umbel inequalities) and the metric invariants that can be derived from them (Markov, diamond, umbel convexity, respectively). The discovery of the asymptotic invariants were inspired by local invariants introduced and/or studied by Eskenazis, S. Li, J. R. Lee, Mendel, Naor, and Peres, and we will crossover between the local and asymptotic facets of the Ribe program.

### Isabelle Chalendar (Université Gustave Eiffel)

### "A note on composition operators on model spaces"

Motivated by the study of composition operators on model spaces launched by Mashreghi and Shabankha we consider the following problem : for a given inner function $\phi \notin \mathsf{Aut}(\mathbb{D})$, find a non-constant inner function $\Psi$ satisfying the functional equation $\Psi \circ \phi = \tau \Psi$, where $\tau$ is a unimodular constant. We prove that this problem has a solution if and only if $\phi$ is of positive hyperbolic step.

Joint work with Pavel Gumenyuk, And John E. MCCarthy.

### Purbayan Chakraborty (Université de Franche-Comté)
### "Schoenberg Correspondence and semigroup of various positive operators"

An unitary error basis on $M_n$ is a generalisation of Pauli matrices which form an orthonormal basis of $M_2$ with respect to the Hilbert-Schmidt inner product. Taking advantage of the isomorphism between $M_n \otimes M_n$, $L(M_n)$ the space of linear maps from $M_n$ to itself and $M_{n^2}$, these unitary error bases are very useful to get a suitable discrete Fourier expansion of a map $T \in L(M_n)$ . These discrete Fourier coefficients which can be thought of as an $n^2 \times n^2$ matrix, give many information about different positivity property of the map $T$. We will discuss a generalised version of the Schoenberg correspondence for non-unital positive semigroup leading to the characterisation of k-(super)positive operators. As an explicit application we will apply our results for the characterisation of all positive semigroup of linear maps on $M_2$ using the Pauli basis and we will also re-establish the famous Lindblad, Kossakowski, Gorini, Sudarshan's(LKGS) theorem in finite dimension characterising the generator of a semigroup of completely positive maps.

### Charles Duquet (Université de Franche-Comté)
### "Unital positive Schur multipliers on $S_n^p$ with a completely isometric dilation"

By Akcoglu's Theorem, we know that all positive contractions on $L^p(\Sigma)$ admit a isometric dilation where $(\Sigma, \mu)$ is a measure space. This result does not extend to non-commutative $L^p$-spaces. It became important to exhibit classes of positive contractions on non-commutative Lp-spaces which do admit an isometric dilation. We will look at Schur Multipliers on $S_n^p$ where $S_n^p$ is the associated Schatten von Neumann class over $n \times n$ matrices. In particular, we will investigate unital positive Schur multipliers which can be dilated into an invertible complete isometry acting on a non-commutative $L^p$-space.

### Micheline Fakhoury (Université d'Artois)
### "Plasticity of the unit ball of some $C(K)$ spaces"

A metric space $(M, \rho)$ is said to be plastic if every non-expansive bijection $F : M \to M$ is in fact an isometry ("non-expansive" means "1-Lipschitz"). In this talk, we aim to explore the plasticity of the unit ball of some $C(K)$ spaces, motivated by the fact that the unit ball of the space $c$ of all convergent sequences of real numbers is plastic. Specifically, we will investigate two cases : when $K$ is a compact metrizable space with a finite number of accumulation points, and more generally, when $K$ is a zero-dimensional compact Hausdorff space with a dense set of isolated points.

### Gilles Godefroy (IMJ-PRG, Sorbonne Université)
### "Explicit copies of $\ell_\infty$ within $\ell_\infty$"

Functional analysis contains several dichotomies which divide a class of objects into "very regular" or "very irregular" ones, without any room left for an intermediate behavior. Baire methods are frequently useful for showing such disjunctions. In this talk, we will consider the following problem : a subspace of $l_\infty$ which is isomorphic to $l_\infty$ is it necessarily weak-star closed or weak-star non Borel ? Several partial results and related examples will be considered, as well as the analogous problem for $L_\infty$.

**Sophie Grivaux (Université de Lille)**

**"Some new results regarding convergence under $\times_q$ of $\times_p$ invariant measures on the circle"**

For each integer $n \geq 1$, denote by $T_n$ the map $x \mapsto nx \mod 1$ from the circle group $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ into itself. Let $p, q \geq 2$ be two multiplicatively independent integers. Using Baire Category arguments, we will show that generically, a continuous $T_p$-invariant probability measure $\mu$ on $\mathbb{T}$ is such that $(T_{q^n}\mu)_{n\geq 0}$ does not converge $w^*$ to the Lebesgue measure on $\mathbb{T}$. This disproves Conjecture (C3) from a 1988 paper by R. Lyons, which is a stronger version of Furstenberg's rigidity conjecture on $\times_p$ and $\times_q$ invariant measures on $\mathbb{T}$, and complements previous results by Johnson and Rudolph. If time permits, I will also present some generalizations of this result concerning convergence to the Lebesgue measure of sequences of the form $(T_{c_n}\mu)_{n\geq 0}$, as well as some extensions to the multidimensional setting.

The talk will be based on a joint work with Catalin Badea (Lille).

**Anders Karlsson (Université de Genève)**

**"A new type of metric fixed point theorem and its applications"**

A general fixed point theorem for isometries in terms of metric functionals is proved under the assumption of the existence of a conical bicombing. This is novel even for Banach spaces and provides a new mean ergodic theorem that in the Hilbert space case implies von Neumann's theorem. For CAT(0)-spaces and injective spaces the fixed point theorem is new for non-locally compact spaces, and implies the usual result for proper CAT(0)-spaces. For Banach spaces the theorem accommodates classically fixed-point-free isometric maps such as those of Kakutani, Edelstein, Alspach and Prus. It also leads to a result in the direction of the invariant subspace problem.

**Tim de Laat**

**"Actions of higher rank groups on uniformly convex Banach spaces"**

I will explain that all affine isometric actions of higher rank simple Lie groups and their lattices on arbitrary uniformly convex Banach spaces have a fixed point. This vastly generalises a recent breakthrough of Oppenheim. Combined with earlier work of Lafforgue and of Liao on strong Banach property (T) for non-Archimedean higher rank simple groups, this confirms a long-standing conjecture of Bader, Furman, Gelander and Monod. As a consequence, we deduce that box space expanders constructed from higher rank lattices are superexpanders.

This is joint work with Mikael de la Salle.

**Colin Petitjean (Université Gustave Eiffel)**

**"On the weak topology in Lipschitz free spaces"**

For a metric space $(M, d)$, the Lipschitz free space (also known as Arens ?Eells space or transportation cost space) $\mathcal{F}(M)$ is a Banach space which is built around $M$ in such a way that $M$ is isometric to a (linearly dense) subset $\delta(M)$ of $\mathcal{F}(M)$, and Lipschitz maps from $\delta(M)$ into any Banach space $X$ uniquely extend to bounded linear operators from $\mathcal{F}(M)$ into $X$. The study of free spaces is at the intersection of functional analysis, metric geometry, measure theory, transportation theory, etc. A recent program consists in trying to characterise (linear) properties of $\mathcal{F}(M)$ in terms of (metric) properties of $M$. In this talk, we will present a few results concerning the weak topology in Lipschitz free spaces.

**Alain Valette (Université de Neuchâtel)**

**"Maximal Haagerup subgroups in $\mathbb{Z}^2 \rtimes GL_2(\mathbb{Z})$ (after Jiang and Skalski)"**

The Haagerup property (a.k.a a-(T)-menability) is a weak form of amenability. In a countable group, every Haagerup subgroup is contained in a maximal Haagerup subgroup, by Zorn's lemma. The study of maximal Haagerup subgroups of a given group was initiated in 2021 by Y. Jiang and A. Skalski, who classified maximal Haagerup subgroups in $\mathbb{Z}^2 \rtimes GL_2(\mathbb{Z})$. By simplifying the original proof we are able to extend it to more general semi-direct products.

**Zhenguo Wei (Université de Franche-Comté)**

**"Schatten class membership of noncommutative martingale paraproducts"**

We study Schatten class membership of semicommutative and purely noncommutative martingale paraproducts, especially for CAR algebras and $\overset{\infty}{\underset{k=1}{\otimes}} \mathbb{M}_d$ in terms of martingale Besov spaces. Using Hytönen's dyadic martingale technique, we also obtain sufficient conditions on the Schatten class membership and the boundedness of operator-valued commutators involving general singular integral operators. In addition, we show the boundedness of commutators still involving general singular integral operators concerning $BMO$ spaces.